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In like manner, place the following numbers as described above along *CB* beginning at *C*.

48, 47, 52
53, 49, 45
46, 57, 50

These numbers, when added horizontally, vertically, or diagonally, give a sum=147.

In the same way, place the following numbers in the unit square along *AB* beginning at *A*.

15, 16, 33, 30, 31
37, 22, 27, 26, 13
36, 29, 25, 21, 14
18, 24, 23, 28, 32
19, 34, 17, 20, 35

These, when added horizontally, vertically, or diagonally, give a sum=125.

Now $4\times46+3\times147=5\times125$, that is, the sum of all numbers in the hypotenuse—square=the sum of all numbers in the two leg-squares.

134. Proposed by J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.

If *ABCD* is a quadrilateral circumscribing a circle, show that the line joining the middle points of the diagonals *AB*, *CD* passes through the center of the circle.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio Univsrsity, Athens, Ohio.

Taking two of the lines as axes and the other two as $lx+my=1$......(1), $l'x+m'y=1$....(2), we find the two vertices of the quadrilateral on the coördinate axes $(0,\ 1/m)$, $(1/l',\ 0)$, and the middle of the diagonal of which these are the extremities $(1/2l',\ 1/2m)$....(3).

The coördinates of the intersection of (1) and (2) are

$$x'=\frac{m-m'}{l'm-lm'},\quad y'=\frac{l-l'}{l'm-lm'}\ldots.(4),$$

which and the origin are the extremities of the second diagonal. The middle of this diagonal is

$$\left(\frac{m-m'}{2(l'm-lm')},\quad \frac{l-l'}{2(l'm-lm')}\right)\ldots.(5).$$

Any conic touching the four lines is of the form

$$(ax+by-1)^2-2\lambda xy=0\ldots.(6).$$

For (6) to be tangent to (1) and (2),

$$\lambda=2(a-l)(b-m)\ldots.(7)\ \text{or}\ \lambda=2(a-l')(b-m')\ldots.(8),$$

respectively. The center of (6) is given by

$$a(2ax-1)-\lambda y=0 \quad \ldots(9).$$

Eliminating a, b, and λ from (7), (8), and (9),

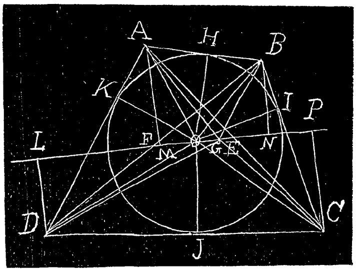
$$2ll'(m'-m)x+2mm'(l'-l)y+(l'm'-lm)=0 \ldots.(10),$$

the locus of the centers of all conics inscribed in the given quadrilateral.

(10) is satisfied by (3) and (5).

II. Solution by **G. B. M. ZERR, A.M., Ph.D.**, Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $ABCD$ be the quadrilateral; circumscribing the circle, center O; E, F the midpoints of AC, BD, respectively. Take G any other point in line LP through EF. Draw DL, AM, BN, CP perpendicular to LP. Then $\angle LDF=\angle NBF$, $DF=BF$.



$\therefore$ $DL=BN$. Also $\angle MAC=\angle PCA$, $AE=CE$.

$\therefore AM=CP$.

$\triangle BGC=\triangle BEC+\triangle BGE+\triangle CGE$.

$\triangle AGD=\triangle AED-\triangle AGE-\triangle GDE$.

But $\triangle BGE=\triangle GDE$, $\triangle CGE=\triangle AGE$.

$\therefore$ $\triangle BGC+\triangle AGD=\triangle BEC+\triangle AED$.

But $\triangle BEC=\frac{1}{2}\triangle ABC$, $\triangle AED=\frac{1}{2}\triangle ADC$.

$\therefore$ $\triangle BGC+\triangle AGD=\frac{1}{2}\triangle ABC+\frac{1}{2}\triangle ADC=\frac{1}{2}$ quadrilateral $ABCD$.

Similarly $\triangle AGB+\triangle DGC=\frac{1}{2}$ quadrilateral $ABCD$.

Therefore the straight line LP through EF is the locus of any point G which moves so that the sum of the triangles BGC and AGD is equal to the sum of the triangles AGB and CGD.

Now $\triangle AOB=\frac{1}{2}r.AB$, $\triangle BOC=\frac{1}{2}r.BC$.

$\triangle COD=\frac{1}{2}r.DC$, $\triangle DOA=\frac{1}{2}r.AD$.

Since $AB+DC=BC+AD$, $\triangle AOB+\triangle DOC=\triangle BOC+\triangle DOA=\frac{1}{2}$ quadrilateral $ABCD$.

$\therefore$ O must be on LP.

NOTE.—This theorem is exercise 66, on page 469, of Phillips & Fisher's Elements of Geometry. We are quite sure that the ordinary Freshman would experience much trouble in effecting a demonstration. Professor Zerr has effected an excellent proof by Euclidean Geometry. The theorem was also proved by *COOPER D. SCHMITT* and *CHARLES C. CROSS*.